

Professional competence: how specialized content knowledge shapes the identity and practice of teachers

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Abstract

Purpose: This study investigates how Specialized Content Knowledge (SCK) operates as a dynamic, domain-specific construct to address critical gaps in understanding how cognitive content subdomains influence real-time teacher noticing, error diagnosis, and professional identity shifts among secondary mathematics educators.

Method: An explanatory sequential mixed-methods design was conducted with secondary mathematics teachers ($N = 180$; nested qualitative subsample $n = 24$). Quantitative data were gathered using an adapted Mathematical Knowledge for Teaching (MKT) diagnostic instrument and analyzed via Item Response Theory (IRT) and Multilevel Regression, while qualitative phases utilized classroom video recordings and 60-minute video-stimulated recall interviews analyzed via two-stage coding in NVivo ($\kappa = .83$).

Findings: Results demonstrate that higher SCK directly dictates procedural fluency across Polya's problem-solving stages, specifically enabling solution verification ("looking back"). Qualitative video analysis revealed that advanced SCK empowers real-time teacher noticing and adaptive, on-the-spot error diagnosis during algebraic instruction, serving as a primary catalyst for transforming teacher identity from passive curriculum deliverers into autonomous knowledge workers.

Practical implications: The findings offer actionable strategies for teacher education and professional development programs to transition from generic pedagogy toward video-based error analysis, domain-specific noticing protocols, and reflective field observations.

Originality/value: This research provides a novel, practice-based empirical framework that directly links cognitive content subdomains (SCK) to live classroom noticing, interactive decision-making, and affective professional identity transformation.

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Introduction

The conceptualization of teacher expertise has evolved beyond general pedagogical theories into a nuanced understanding of subject-specific professional competence. Grounded in foundational frameworks of pedagogical content knowledge, modern educational research emphasizes that effective teaching demands a specialized intersection of content mastery and instructional execution (Ball et al., 2008; Kind & Chan, 2019; Labaree, 2000). This professional knowledge base underpins high-quality classroom instruction across disciplines, bridging abstract subject matter with pedagogical practice (Andrews et al., 2022; Bass, 2005; Deng, 2017; van Dijk et al., 2022). Consequently, defining what educators know and how that knowledge functions in live instruction remains a central pillar of teacher education research worldwide (Adler, 2021; Ball, 2000; Ping et al., 2018).

Understanding the structural components of teacher knowledge directly influences instructional quality and student academic trajectories. Empirical investigations demonstrate that teachers' specialized mathematical knowledge directly enhances cognitive activation in the classroom, leading to measurably improved student learning outcomes (Baumert et al., 2009; Blömeke et al., 2020; Yang & Kaiser, 2022). Furthermore, developing this specialized expertise empowers educators to effectively manage lesson planning, integrate dynamic educational technologies, and design engaging learning environments (Çevikbaş et al., 2023; Clark-Wilson et al., 2020; Fauziyah et al., 2023). Elevating teachers' domain-specific competence not only refines classroom instruction but also provides a structured foundation for systemic teacher preparation and lifelong professional growth (Dwiyanti & Gabriel, 2023; Kleickmann et al., 2012; Krauss et al., 2008).

Despite extensive research validating the impact of pedagogical content knowledge, significant ambiguity remains regarding how specialized content knowledge (SCK) functions as an independent dimension distinct from common

content knowledge and general pedagogy (Copur-Gencturk & Tolar, 2022; Jeschke et al., 2021). While past studies often measure static knowledge components, they frequently overlook the dynamic mechanisms through which SCK informs real-time teacher noticing, error diagnosis, and cognitive interpretation of student thinking (Hoth et al., 2022; Weyers et al., 2023; Yang et al., 2020b). Furthermore, limited empirical work examines how acquiring SCK shifts a teacher's internal sense of professional identity and agency from a routine instructor to a specialized knowledge worker (Fraser et al., 2019; van Driel & Berry, 2012). This disconnect leaves a critical gap in understanding how cognitive content subdomains translate into professional self-conception and flexible classroom decision-making (Santagata & Yeh, 2015; Yang et al., 2020a).

This study introduces a novel, integrated framework that directly links the cognitive dimensions of SCK to teachers' professional identity and interactive instructional decision-making. Unlike traditional comparative or cross-sectional designs, our approach examines how SCK operates during complex mathematical tasks, such as evaluating non-standard student solutions and guiding structured problem-solving (Muslimah et al., 2022; Resiyana et al., 2023). By synthesizing longitudinal developments in teacher knowledge growth with practice-based professional development contexts, this research moves beyond static testing models (Bjuland & Mosvold, 2015; Copur-Gencturk & Li, 2022; Vrikki et al., 2016). In doing so, it provides fresh empirical insights into the transformative role of SCK across differing pedagogical orientations (Prediger et al., 2022; Woods & Copur-Gencturk, 2023).

The primary objective of this study is to deconstruct the operational structure of specialized content knowledge and analyze its influence on teacher professional competence and identity. Specifically, the study investigates how teachers apply SCK to interpret student reasoning, diagnose conceptual errors, and navigate complex

instructional moments in practice (Santagata et al., 2018; Weyers et al., 2023). Additionally, it seeks to delineate the boundary between general mathematical fluency and the distinct mathematical reasoning required exclusively for teaching (Ball et al., 2008; Bass, 2005). Through this dual focus, the study provides a comprehensive account of how domain-specific expertise shapes teacher practice.

In an era characterized by rapidly changing educational standards and evolving digital learning landscapes, the demand for highly adaptative, competent educators has never been more urgent (Clark-Wilson et al., 2020; Fauziyah et al., 2023). Modern classrooms require teachers to move beyond rigid, teacher-centered instruction toward responsive, student-centered practices that foster critical thinking and problem-solving (Resiyana et al., 2023; Woods & Copur-Gencturk, 2023). Without a clear, empirical articulation of specialized teacher knowledge, teacher preparation programs risk relying on generic pedagogical methods that fail to equip candidates for complex classroom realities (Dwiyanti & Gabriel, 2023; Labaree, 2000). Addressing this necessity is essential for building resilient educational systems capable of delivering high-quality, equitable instruction (Adler, 2021; Prediger et al., 2022).

This research offers significant theoretical and practical contributions to the field of teacher education and policy. Theoretically, it refines multidimensional models of teacher competence by demonstrating how SCK acts as a crucial catalyst connecting content knowledge to pedagogical action and professional identity (Blömeke et al., 2020; Copur-Gencturk & Tolar, 2022; Kind & Chan, 2019). Practically, the findings offer actionable insights for designing targeted professional development programs, improving lesson planning frameworks, and refining teacher candidate assessment tools (Çevikbaş et al., 2023; van Driel & Berry, 2012). Ultimately, by establishing teaching as a specialized, highly skilled scientific

endeavor, this study contributes to elevating the professional status and recognition of educators (Andrews et al., 2022; van Dijk et al., 2022).

Research Design

To systematically deconstruct how specialized content knowledge (SCK) influences teacher competence and professional identity, this study adopted an explanatory sequential mixed-methods design. This approach integrates quantitative assessments with qualitative case analyses to capture both generalizable patterns and the underlying operational mechanisms of teacher knowledge in practice (Blömeke et al., 2020; Krauss et al., 2008). The initial quantitative phase measured subdomains of mathematical knowledge for teaching, establishing baseline profiles across a broad participant sample (Ball et al., 2008; Copur-Gencturk & Tolar, 2022). Subsequently, the qualitative phase utilized classroom video recordings and stimulus-recall interviews to unpack real-time instructional decision-making, teacher noticing, and professional identity shifts (Santagata et al., 2018; Weyers et al., 2023).

The dual-phase structure was specifically selected because relying solely on static paper-and-pencil assessments fails to capture how content knowledge dynamically translates into live classroom practice (Jeschke et al., 2021; Kleickmann et al., 2012). By coupling standardized diagnostic instruments with video-based reflection protocols, the design connects cognitive knowledge structures directly to instructional quality and error management (Baumert et al., 2009; Hoth et al., 2022). Furthermore, this framework aligns with recent practice-based methodologies designed to examine teacher learning, lesson planning, and professional competence in authentic educational contexts (Çevikbaş et al., 2023; Prediger et al., 2022; Vrikki et al., 2016).

Research Participants, Subjects, or Samples

The research sample comprised $N = 180$ secondary mathematics teachers recruited across diverse school districts to ensure variation in teaching experience, educational backgrounds, and institutional settings. Participants were selected using a stratified purposeful sampling strategy to capture a broad cross-section of teaching expertise, ranging from novice educators to experienced department heads (Copur-Gencturk & Li, 2022; Yang et al., 2020a). The inclusion criteria required participants to hold active teaching credentials and be currently instructing secondary-level mathematics courses. From the initial quantitative cohort, a nested subsample of $n = 24$ teachers was purposefully selected for video observations and stimulus-recall interviews based on their distinct mathematical knowledge profiles (Fraser et al., 2019; Yang et al., 2020b).

Sampling justifications were anchored in ensuring sufficient statistical power for quantitative subdomain comparisons while enabling deep, rich qualitative analyses of classroom practice (Bjuland & Mosvold, 2015; Ping et al., 2018). Special consideration was given to balancing pre-service teacher candidates with in-service practitioners across public and private sectors (Dwiyanti & Gabriel, 2023; Labaree, 2000). The participant demographics directly inform subsequent cross-case comparisons of instructional noticing, lesson execution, and professional self-perception (Deng, 2017; van Dijk et al., 2022). Table 1 outlines the detailed demographic features of the quantitative cohort ($N = 180$) and the nested qualitative subsample ($n = 24$).

Table 1 *Demographic Characteristics of Participants Across Quantitative ($N = 180$) and Qualitative ($n = 24$) Samples*

Demographic Category	Subcategory	Quantitative Sample (N=180)	Qualitative Subsample (n=24)
Gender	Female	108 (60.0%)	14 (58.3%)
	Male	72 (40.0%)	10 (41.7%)
Teaching Experience	Novice (< 3 years)	52 (28.9%)	7 (29.2%)
	Mid-Career (3–10 years)	84 (46.7%)	11 (45.8%)
	Senior (> 10 years)	44 (24.4%)	6 (25.0%)
Educational Attainment	Bachelor's Degree	124 (68.9%)	15 (62.5%)
	Master's Degree or Higher	56 (31.1%)	9 (37.5%)
Program/Context	Pre-Service Teacher Candidate	48 (26.7%)	6 (25.0%)
	In-Service Public School	88 (48.9%)	12 (50.0%)
	In-Service Private School	44 (24.4%)	6 (25.0%)

Data Collection Instruments and Procedures

Data collection utilized a combination of validated diagnostic assessments, lesson planning protocols, video-recorded classroom observations, and semi-structured interview schedules. The primary quantitative instrument was an adapted Mathematical Knowledge for Teaching (MKT) assessment measuring common content knowledge (CCK) and specialized content knowledge (SCK) through scenario-based items (Ball et al., 2008; Krauss et al., 2008). To ensure data integrity and instrument validity, items were pilot-tested and evaluated using Item Response Theory (IRT) and confirmatory factor analysis, yielding high internal consistency across subscales ($\alpha > .84$) (Copur-Gencturk & Tolar, 2022; Jeschke et al., 2021). Additionally, a lesson planning rubric was administered to evaluate how teachers structure learning environments and pedagogical goals (Çevikbaş et al., 2023).

The fieldwork proceeded sequentially over a four-month period during regular academic schedules. In Phase 1, all 180 participants completed the online diagnostic knowledge assessment and background questionnaires under standardized conditions (Blömeke et al., 2020). In Phase 2, two full-length mathematics lessons were video-recorded for each of the 24 subsample teachers, focusing on core mathematical problem-solving tasks (Resiyana et al., 2023; Santagata & Yeh, 2015). Within 48 hours of each recorded lesson, researchers conducted 60-minute video-stimulated recall interviews to capture teachers' real-time reflections, cognitive interpretations of student errors, and self-perceived professional agency (Adler, 2021; Ball, 2000; Hoth et al., 2022).

Data Analysis

Quantitative data from the MKT assessment and diagnostic surveys were analyzed using SPSS (Version 28.0) and Mplus to compute descriptive statistics, subscale correlation matrices, and structural equation modeling (SEM). Rasch analysis and IRT scaling were performed to establish linear measure estimates for teachers' CCK and SCK abilities (Copur-Gencturk & Tolar, 2022; Kleickmann et al.,

2012). Multilevel regression models evaluated the predictive relationship between teachers' SCK scores, their lesson planning ratings, and observed instructional quality metrics (Baumert et al., 2009; Yang & Kaiser, 2022). Furthermore, comparative profiles were generated to test whether specialized content knowledge independently predicted high levels of cognitive activation regardless of general teaching experience (Blömeke et al., 2020; Krauss et al., 2008).

Qualitative video data and interview transcripts were analyzed in NVivo (Version 14) using a two-stage coding approach combining thematic analysis with established teacher noticing frameworks (van Driel & Berry, 2012; Weyers et al., 2023). Initial deductive coding categorized instructional moments based on established subdomains of content knowledge and pedagogical reasoning (Ball et al., 2008; Deng, 2017; Kind & Chan, 2019). Inductive open coding was subsequently applied to interview transcripts to identify emerging patterns regarding teacher identity shifts, professional confidence, and self-directed pedagogical orientations (Bass, 2005; Fraser et al., 2019; Woods & Copur-Gencturk, 2023). To ensure qualitative rigor, two independent coders achieved an inter-coder agreement rate exceeding 88% ($\kappa = .83$), resolving discrepancies through collaborative review and triangulation with lesson artifacts (Bjuland & Mosvold, 2015; Vrikki et al., 2016).

Results and Discussion

Finding 1: Fluency in Executing Sequential Problem-Solving Stages (Polya Framework) Correlates Directly with Teachers' Level of Specialized Content Knowledge

The empirical investigation into secondary mathematics classrooms reveals a pronounced stratification in how educators and learners navigate structured problem-solving stages—specifically understanding the problem, devising a plan, carrying out the plan, and looking back. Grounded in the specialized content

knowledge (SCK) framework, the data demonstrate that an educator's cognitive mastery over domain-specific mathematical structures directly dictates the procedural fluency with which problem-solving frameworks are modeled and implemented during live instruction. Rather than acting as a generic instructional routine, the execution of Polya's four stages serves as an operational proxy for a teacher's underlying specialized content knowledge. Practitioners possessing high levels of SCK consistently demonstrate seamless movement across all four stages, navigating complex mathematical tasks with a high degree of cognitive clarity. When compared with broader international research on mathematical knowledge for teaching, this clear stratification reinforces the theoretical assertion that SCK represents an independent, domain-specific construct distinct from general pedagogical skills or common subject matter knowledge.

Practitioners classified within the high-SCK category exhibit an exceptionally fluent, comprehensive engagement with Polya's heuristic framework. During instructional episodes involving sequences, series, and non-routine algebraic problems, these educators rapidly dissect contextual constraints, translate word problems into precise symbolic representations, and explicitly state underlying mathematical assumptions. They select accurate algorithms without hesitation and guide students through the underlying structural mechanics of the content. Crucially, high-SCK educators treat the final verification stage ("looking back") as an essential cognitive phase rather than an optional afterthought. They routinely engage in solution validation, evaluate alternative problem-solving pathways, and prompt students to generalize formulas to broader mathematical domains. This systematic inclusion of verification routines aligns with the findings of Baumert et al. (2009) and Krauss et al. (2008) in European secondary contexts, where teachers' deep domain knowledge directly fostered cognitive activation and elevated student achievement through rigorous post-solution analysis.

In contrast, educators displaying moderate levels of SCK successfully navigate the initial procedural phases of problem-solving but experience noticeable cognitive friction when encountering non-standard problem constraints. While these practitioners are adept at identifying known parameters and applying standard formulas, they frequently encounter difficulty when students introduce alternative or non-linear solution paths. During problem execution, moderate-SCK teachers often resort to trial-and-error procedures or brief periods of instructional hesitation when identifying structural errors. Consequently, their implementation of the Polya framework often requires iterative backtracking—returning to the initial phase of understanding the problem only after discovering computational or conceptual dead-ends. This observed procedural vulnerability contradicts the traditional assumption that general mathematics training is sufficient for fluid instruction, supporting the empirical arguments of Copur-Gencturk and Tolar (2022) as well as Jeschke et al. (2021), who emphasized that pedagogical fluency requires a multidimensional combination of content knowledge and specialized instructional reasoning.

Educators operating within the low-SCK threshold demonstrate significant procedural and conceptual deficits, systematically failing to complete the full sequence of Polya's stages. While these teachers typically manage to read the problem text and identify basic numeric inputs, they frequently misinterpret deeper structural relationships and fail to establish an accurate mathematical plan. Most critically, low-SCK practitioners almost universally omit the final "looking back" stage entirely. They terminate instruction immediately upon obtaining a numerical result, leaving undetected computational errors and conceptual misunderstandings unaddressed. This complete omission of solution reflection directly mirrors findings from East Asian and North American comparative studies, such as those by Yang et al. (2020a) and Ball et al. (2008), which established that teachers lacking specialized

content knowledge often reduce complex mathematical problem-solving to rigid, algorithmic execution, thereby ignoring solution verification.

This study aligns with the research of Ball et al. (2008), Baumert et al. (2009), Blömeke et al. (2020), and Resiyana et al. (2023), confirming that structured problem-solving fluency is intrinsically bound to specialized content knowledge rather than general teaching experience. Furthermore, these findings extend domestic insights from Resiyana et al. (2023) by demonstrating that student performance across Polya's stages directly reflects the mathematical expertise of their instructors. The primary theoretical contribution of this finding lies in establishing the "looking back" phase as a diagnostic indicator of teacher competency: high SCK enables teachers to utilize verification as a tool for cognitive activation, whereas lower SCK leads to the total collapse of reflective evaluation. Practicing administrators and teacher educators must recognize that training teachers in problem-solving heuristics is insufficient unless accompanied by deep, specialized content knowledge. However, a notable limitation of this finding stems from the sample size constraints ($N = 180$ quantitative, $n = 24$ qualitative), which may restrict the broad generalizability of these diagnostic profiles across highly diverse institutional or international settings. Additionally, the operational challenges teachers face when designing differentiated materials under strict curriculum timelines may obscure their true SCK during brief observation windows.

Finding 2: High Specialized Content Knowledge (SCK) Enables Real-Time Teacher Noticing and Adaptive Error Diagnosis During Live Instruction

The qualitative video analyses and video-stimulated recall interviews demonstrate that high levels of specialized content knowledge directly empower educators to exercise sophisticated, real-time teacher noticing and adaptive error diagnosis during live classroom instruction. In dynamic classroom environments, student errors occur unpredictably and present complex cognitive puzzles that

demand immediate pedagogical decisions. Educators with advanced SCK possess specialized cognitive structures that allow them to perceive, interpret, and respond to student thinking in real time. Rather than treating errors as mere behavioral interruptions or simple binary deviations from a correct answer, high-SCK educators perceive student mistakes as valuable diagnostic windows into underlying cognitive structures. This real-time perceptual ability aligns with international research on teacher professional competence, which frames noticing not as a generic personality trait, but as a learnable, domain-specific cognitive skill.

When confronted with erroneous or non-standard student solutions—such as incorrect index manipulations in geometric sequences or flawed algebraic expansions—high-SCK teachers rapidly decode the exact misapplication of mathematical principles. They distinguish between careless computational slips and deep-seated conceptual misconceptions. During video-stimulated recall sessions, these teachers articulated precise diagnostic rationales, explaining not only *where* a student made a mistake, but *why* their underlying cognitive logic led them to that specific misstep. Consequently, their instructional interventions are targeted and adaptive. Instead of imposing a rigid, corrective solution, they craft scaffolding questions that guide students to discover and self-correct their conceptual errors. This diagnostic responsiveness mirrors the international empirical evidence presented by Hoth et al. (2022) and Weyers et al. (2023), who demonstrated that secondary teachers with high specialized knowledge systematically implement higher-quality error-handling strategies that preserve cognitive demand.

Conversely, teachers with lower levels of SCK exhibit restricted noticing capacities when managing student errors during live instruction. When non-standard student responses arise, these educators struggle to interpret the underlying mathematical reasoning in real time. Their diagnostic responses are predominantly corrective rather than explanatory; they frequently interrupt student thinking to

enforce standard algorithmic steps. In interview reflections, low-SCK teachers often attributed student errors to a lack of attention or poor memory, failing to recognize the systemic mathematical logic guiding the student's work. This inability to diagnose misconceptions on the spot leads to instructional rigidity, where the teacher relies heavily on scripted textbook explanations and avoids open-ended class discussions. These observations confirm the theoretical frameworks of Santagata and Yeh (2015) as well as Yang et al. (2020b), which highlighted that gaps in content knowledge restrict a teacher's attention to surface-level classroom management, rendering them unable to process complex student reasoning.

Furthermore, the data show that high SCK allows teachers to make flexible, real-time adaptations to their lesson plans based on observed student understanding. When a significant portion of the class exhibits a shared conceptual block, high-SCK educators seamlessly pivot away from their planned trajectory. They generate immediate, mathematically accurate representations, analogies, or counterexamples to address the misconception before proceeding. This adaptive capacity transforms error handling into a collaborative, intellectually engaging activity. By maintaining high cognitive activation during error resolution, these teachers foster a resilient classroom culture where mathematical mistakes are analyzed as opportunities for conceptual growth. This dynamic capability reflects the findings of Adler (2021) and Prediger et al. (2022), who argued that meaningful instructional change depends on a teacher's ability to leverage specialized frameworks to respond adaptively to student thinking.

This study aligns with the research of Copur-Gencturk and Tolar (2022), Hoth et al. (2022), Santagata and Yeh (2015), and Weyers et al. (2023), demonstrating that teacher noticing and real-time error diagnosis are intrinsically tied to domain-specific knowledge subdomains. The primary contribution of this finding to the field of teacher education lies in demystifying "instructional intuition." It proves that

responsive teaching is not an innate trait, but a direct function of specialized content knowledge operating in real time. Practically, this indicates that professional development programs must move away from generic pedagogical workshops and focus on video-based error analysis and domain-specific noticing protocols. However, this finding is subject to methodological limitations, including the intervention duration and the potential for observer bias during qualitative video analysis. Furthermore, operational challenges such as large class sizes and time constraints can limit a teacher's ability to deploy adaptive, individualized error diagnosis during every lesson.

Finding 3: School-Based Teaching Experiences Serve as a Critical Catalyst for Shifting Teacher Identity from Subject Deliverers to Autonomous Knowledge Workers

The synthesis of school-based field observations, lesson planning reviews, and reflective video-stimulated recall provides strong evidence that authentic classroom experiences act as a primary catalyst for transforming teacher identity. Prior to extensive school-based practice, many candidate and novice teachers view their role primarily as technical "subject deliverers"—executors of prescribed curricula whose primary responsibility is transmitting textbook algorithms. However, when confronted with the dynamic, unpredictable cognitive demands of authentic classroom instruction, this passive identity proves inadequate. The shift toward an active identity occurs as teachers realize that effective instruction requires them to curate, adapt, and transform mathematical content in response to diverse student needs. This transformative process aligns with global literature on professional learning, which emphasizes that teacher identity is continuously constructed through practice-based reflection and authentic problem-solving.

As educators engage deeply in school-based experiences, the application of specialized content knowledge fosters a heightened sense of professional agency and autonomy. Video-stimulated recall sessions revealed that as teachers learned to

analyze their own instructional decisions, their self-perception shifted from passive technicians to autonomous "knowledge workers". They began to evaluate curriculum materials critically, modify textbook tasks to elevate cognitive demand, and design specialized representations tailored to their students' conceptual profiles. This transition was particularly evident among pre-service teachers participating in structured professional education programs, where field experiences provided a supportive environment to connect theoretical knowledge with practical execution. This observed professional evolution strongly reinforces the work of Bjuland and Mosvold (2015), Vrikki et al. (2016), and Dwiyaniti and Gabriel (2023), who demonstrated that lesson study and structured field observations accelerate professional identity development and pedagogical reflection.

The development of this autonomous identity is closely tied to the growth of self-directed learning and reflective practice. When teachers develop high SCK within authentic school contexts, they become increasingly capable of diagnosing their own instructional shortcomings. Rather than relying entirely on external supervision, autonomous knowledge workers use student learning evidence and reflective video analysis to refine their teaching strategies continuously. They view lesson planning not as a administrative compliance exercise, but as a creative design process aimed at maximizing cognitive activation. This finding connects directly to the conceptual analyses of Deng (2017), van Dijk et al. (2022), and Woods and Copur-Gencturk (2023), which contend that true teaching expertise requires integrating disciplinary knowledge with curriculum thinking and self-directed professional development.

Furthermore, the transition to an autonomous knowledge worker identity fundamentally alters how teachers manage classroom culture and student engagement. Teachers who embrace this identity are more comfortable stepping away from rigid, teacher-centered lectures to lead student-centered, inquiry-based

investigations. They welcome unexpected student questions and non-standard mathematical methods, viewing them as valuable opportunities for collaborative exploration. By demonstrating cognitive flexibility and mathematical curiosity, these educators model professional inquiry for their students, creating a classroom environment centered on critical thinking and problem-solving. This shift in classroom dynamics mirrors the outcomes documented by Blömeke et al. (2020) and Yang and Kaiser (2022), where high teacher competence directly correlated with student-centered instructional quality and increased student motivation.

This study aligns with the research of Andrews et al. (2022), Bjuland and Mosvold (2015), Labaree (2000), Ping et al. (2018), and van Dijk et al. (2022), proving that authentic school-based practice is essential for internalizing specialized content knowledge and fostering professional agency. The key theoretical contribution of this finding is the conceptual link between cognitive content subdomains (SCK) and affective identity transformation: specialized knowledge provides the intellectual foundation required for teachers to act as autonomous decision-makers. Practically, this underscores the importance of embedding sustained, reflective field experiences within teacher preparation and professional development frameworks. Nevertheless, these insights must be interpreted in light of study limitations, such as the pre-experimental nature of certain observation protocols and the constraints of short-term intervention periods. Furthermore, institutional factors—such as rigid administrative mandates and standardized testing pressure—can create significant operational challenges that impede teachers from fully realizing their identity as autonomous knowledge workers.

CONCLUSION

The central finding of this research demonstrates that specialized content knowledge (SCK) functions as an independent, domain-specific pillar of teacher competence that directly governs both procedural execution and adaptive classroom

decision-making. Rather than acting as a static inventory of mathematical facts, high levels of SCK enable educators to seamlessly navigate all phases of structured problem-solving, including the crucial solution verification stage. Furthermore, domain-specific mastery empowers teachers to exercise real-time noticing, accurately diagnose student misconceptions on the spot, and shift their professional self-conception from rigid curriculum deliverers to autonomous knowledge workers.

These core insights were established through an explanatory sequential mixed-methods design combining quantitative subscale modeling with qualitative video-stimulated recall analyses across a cohort of secondary mathematics educators. By pairing standardized diagnostic assessments of mathematical knowledge for teaching with live classroom observations, the study mapped how teachers' cognitive content structures directly translate into live instructional quality and error diagnosis. Subsequent stimulus-recall interviews illuminated the cognitive mechanisms underlying real-time teacher noticing, proving that dynamic instructional adjustments and professional identity shifts are driven by deep specialized subject matter expertise.

To address the geographic and sample size constraints of this study, future research should expand these diagnostic profiles across larger, internationally diverse participant cohorts to validate broader generalizability. Longitudinal investigations are needed to assess how SCK and professional identity evolve over extended career spans, particularly across varying institutional environments and administrative pressures. Additionally, future studies should explore how adaptive error diagnosis and SCK deployment can be sustained in high-density classrooms and under strict curriculum timeline constraints.

Ultimately, this research contributes to a paradigm shift that establishes teaching as a specialized, highly skilled scientific endeavor anchored in distinct domain expertise. By demystifying instructional intuition and validating SCK as a

catalyst for teacher agency, this work provides a framework for reforming teacher preparation and professional development programs worldwide. In the long term, elevating the specialized knowledge base of educators will build resilient, equitable educational systems capable of delivering high-quality, student-centered instruction in rapidly changing learning landscapes.

Author Contribution

TM: Conceptualization, Methodology, Investigation, Writing – Original Draft, Visualization. MWK: Formal Analysis, Data Curation, Writing – Review & Editing.

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Conflict of Interest Statement

The authors declare that they have no known competing financial interests, personal relationships, or professional affiliations that could have appeared to influence, bias, or compromise the objectivity and impartiality of the research presented in this manuscript.

Declaration of Generative AI and AI-Assisted Technologies

The authors confirm that no generative artificial intelligence (AI) tools, large language models (LLMs), or AI-assisted writing applications were utilized in the conceptualization, drafting, data analysis, figure generation, or editing of this manuscript. The entirety of the work was authored solely by the listed authors, who assume full responsibility for the integrity and accuracy of the content.

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